Uppsala University
Department for Information Science
Statistics

B1:1

2008-02-21

WRITTEN EXAM FOR THE COURSE, *PROBABILITY THEORY AND INFERENCE THEORY*, B1:1 (7.5 ECTS) 2008-02-21.

Writing time: 8^{00}-12^{00}

Permitted aids: Mendenhall, Wackerly and Scheaffer: *Mathematical Statistics with Applications*
Sydsaeter: *Matematisk analys för ekonomer* or an equivalent mathematical textbook
Pocket calculator (non-programmable)
Dictionary (or word-list)

The written examination has 5 problems, for a total of 60 points.

If you desire clarification regarding the test, especially the wording of a problem, then please alert an examination proctor. The examination proctors can contact the responsible instructor.

For each problem, the maximum number of points is indicated. When one subproblem depends on the preceding subproblem(s), then points may be earned for the dependent subproblem when the preceding subproblem(s) have been correctly solved.

After turning in your test, you may keep the test-pages with the question-statements.

INSTRUCTIONS:

A. Carefully follow the instructions that are listed on the examination-directions page.

B. State the assumptions that must be made for the method to be applicable.

C. Account for every essential step in your solution. If special concerns are raised in the problem statement, then your solution must carefully address those concerns.

1. (10p) Bowl B_1 contains 2 white chips, bowl B_2 contains 2 red chips, bowl B_3 contains 2 white and 2 red chips, and bowl B_4 contains 3 white and 1 red chips. The probabilities of selecting bowl B_1, B_2, B_3 or B_4 are 1/2, 1/4, 1/8, and 1/8, respectively. A bowl is selected using these probabilities, and a chip is then drawn at random.

a) Find P(W), the probability of drawing a white chip (W denote white color).
b) Find $P(B_1|W)$, the conditional probability that bowl B_1 had been selected, given that a white chip was drawn.

2. (15p) Let Y_1 and Y_2 have the joint probability density function give by

$$f(y_1, y_2) = \begin{cases} 6(1-y_2), & 0 \le y_1 \le y_2 \le 1 \\ 0. & \text{elsewhere.} \end{cases}$$

a) Find marginal density functions of Y_1 and Y_2.
b) Are Y_1 and Y_2 independent?
c) Find $E(Y_1 - 3Y_2)$.

3. (15p) Assume that $X_1, X_2, \ldots, X_{n_1}$ and $Y_1, Y_2, \ldots, Y_{n_2}$ are independent samples from a population with a parameter P . Two unbiased estimators $\hat{p}_1$ and $\hat{p}_2$ with variances $\frac{p(1-p)}{n_1}$ and $\frac{p(1-p)}{n_2}$ are derived based on these two samples respectively. Consider the following estimators of P

$$p_1^* = \frac{\hat{p}_1 + \hat{p}_2}{2} \text{ and } p_2^* = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}.$$

a) show that both estimators are unbiased,
b) derive the variances for the two estimators,
c) Which estimator is more efficient? Hint: Take the difference between the two variances.

4. (10p) Lifetime of a certain type of bulb is exponential distributed with parameter β. A sample of n bulbs is randomly selected and the life time of each of these bulbs are denoted by X. Find Maximum likelihood estimator for β.

5. (10p) A chemical process has produced, on average, 800 tons of chemical per day. The daily yields for the pass week are 785, 805, 790, 793, and 802 tons.

a) Do these data indicate that the average yield is less than 800 tons and hence that some thing is wrong with the process? Test at the 5% level of significance.
b) What assumptions must be satisfied in order for the procedure you used to analyze these data to be valid?
c) Give bounds for the associated p-value.