Supplementary Written Examination in Time Series Analysis (B3)[1]

Spring 2017

2017-05-24 14.00-18.00

Venue: Bergsbrunnagatan 15, room 2

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Allowed means of assistance:

1. Pen or **pencil** (recommended) and eraser
2. **Calculators**,
   (a) 'programmable' calculator, e.g. calculator with graphing functions is OK.
   (b) Calculators with blue-tooth are not allowed.
   (c) Calculators with access to internet are not allowed.
   (d) Calcuators with which it is possible to send and recieve messages of any kind are not allowed.
3. **Physical (paper) dictionary** (no electronic dictionary allowed).
   (a) Dictionary must contain *no notes* of any kind.
   (b) Each student must have his/her own dictionary. It is not allowed for students to pass a dictionary between them.
4. **Ruler.**

[1]Disclaimer: Any narrative in this exam is designed to make the exam tasks to become a bit more 'interesting' by vaguely resemble a potential real life situation for a statistician working in the private or public sector.

Any names, persons, places and events depicted in this exam are purely fictional. All and any potential resemblance or similarities with real persons, places, events or situations is completely coincidental.

No implications whatsoever should be made and no conclusions whatsoever should be drawn regarding any potential political, philosophical or religious views or preferences on the behalf of any person or legal entity involved in the creation of this document.

5. Collection of formulae and Statistical Tables named '*Collection of Formulae and Statistical Tables for the B2-Econometrics and B3-Time Series Analysis courses and exams*', that the student brings to the exam location. The collection must contain no notes whatsoever. This implies that you cannot make any notes during the exam. If checked, any notes in the collection will be assumed to have been made beforehand, and thus subject to investigation concerning deceptive conduct in examination (cheating).

6. Please note that a collection of critical values for the Student´s t, Normal, Chi-square and F-distributions is given in the Appendix of the '*Collection of Formulae and Statistical Tables for the B2-Econometrics and B3-Time Series Analysis courses and exams*'.

7. About degrees of freedom in tests: If, by any chance, the degree of freedom number that you need for a critical value is not in the table, say that you need 125, but there is only 120 and 130 in the table, then choose the lower number of degrees of freedom, that is, in this case 120.

8. Also note that the '*Test template*', that should be used when performing tests, is given in the '*Collection of Formulae and Statistical Tables for the B2-Econometrics and B3-Time Series Analysis courses and exams'.*

That is:

1. NO BOOK (except paper-dictionary) is allowed.

2. NO (student-written) notes are allowed.

3. NO other document than the one 'Collection of Formulae and Statistical Tables for Time Series Exam' is allowed.

Instructions: **Please note the following:**

1. Start with reading through the instructions!

2. Make sure you **follow** the instructions!

3. Start with reading through the exam.

4. You may write your solutions in Swedish or English.

5. If you find something unclear or if you suspect a typo/mistake in any of the tasks - please do not hesitate to contact the staff at the exam-location for them to get in touch with the responsible teacher.

6. Total score is **100** points

(a) If you want the ECTS grades, please indicate that on the cover page!

(b) For each task the maximum number of points is given within parenthesis, e.g. (16p in total).

(c) For each subtask the number of points is given within parenthesis, e.g. (2p)

7. All solutions must be on separate sheets. No solutions on the questionnaire! (If so, they will be disregarded.)

8. Make sure your solutions are: easy to read and easy to understand, that is:

(a) For each task that you solve, please start with a new sheet: after Task 1, start with a blank sheet for Task 2, etc.

(b) Write the *task number* at the top of each page, in the

.................................**MIDDLE OF THE PAGE!!!**...

Like:

....**TASK 1**..

- if you write it in the upper left corner, the staple will cover it, and there is no for way for the examinator to know if the text of that sheet belongs to the previous sub-task or what it is. The Examinators will not make any 'qualified guesses' of what is being displayed on any given page. It is the responsibility of the student to make sure that every task and sub-task is easily identifiable.

(c) If you continue a sub-task on the next sheet of paper - indicate that at the top of the page - IN THE MIDDLE OF THE PAGE, like, for example:

....'Task 1B (cont.)'..

(d) Please separate each subtask A, B etc with a horizontal line across the sheet

____
if they are on the same sheet of paper - that way it will be easy for the examinator to actually see where one subtask ends and next begins.

(e) For examinator readability, it is highly recommended that you use a pencil, (and not a pen), which will allow you to erase and rewrite if you make a mistake. Crossed-over text and corrections using 'tipp-ex' will just cause blurriness and confusion to the examinator.

(f) For examinator readability: Write clearly, that is, letters, mathematical/statistical symbols and numbers should be easy recognizable!! Do not underestimate the correlation between readability and points scored, that is, when readability goes to zero, points scored also goes to zero, no matter your intentions or wheather *you* can read it or not.

(g) Also note that everything that you write will be taken at 'face value'. That is, for example, if you write β_1 the examinator will take that as a β_1 even though you may claim that it is given from the context it should be clear that you meant something else, like β_3. Thus, given this example, writing β_1, and that is not correct in that specific formula or statement, this will lead to subtraction of points, even if you will claim that it is just a typo, and that in another task or subtask, it is clear that you understand the issue.

(h) Please put the sheets in **order**, that is first Task 1, and then Task 2 etc...

9. Please keep the questionaire.

10. Do well!

Task 1

(31 points in total)

A) (4p) This question is in two parts:

1. What is a stochastic process from a theoretical point of view? Explain using words, no formulae needed.
2. What is the conceptual difference between a stochastic process and a realization?

B) (6p) State the conditions for a stochastic process to be *covariance stationary*. For each condition, state that condition using formulae and also explain in words what it means.

To 'apply' the Box-Jenkins methodology, a nessecary condition is that the series in question is (at least) covariance stationary. If a process is *not* stationary, we need to transform it somehow to make it stationary before we can apply the Box-Jenkins methodology.

C) (4p) Write down a model that is *difference* stationary. Suggest a tranformation that makes it stationary.

D) (4p) Write down a model that is *trend* stationary. Suggest a tranformation that makes it stationary.

E) (8p) State the four stages of the Box-Jenkins methodology. For each stage, elaborate on the *purpose* of that specific stage, also give at least *one* example of a tool/method/statistical test that can be used in that specific stage.

F) (3p) Explain what it means that two variables are cointegrated. State the nessecary conditions for two variables in terms of order of integration.

G) (2p) Explain the conceptual difference between the ARMA class of models and the GARCH class of models. What is being modelled by what class of models?

Task 2

(17 points in total)

Consider the following process

$$\phi(B) Y_t = \theta(B) e_t$$

where $e_t \sim NID(0, \sigma^2)$.

Let

$$\phi(B) = (1 - B\phi)$$
$$\theta(B) = (1 - B\theta),$$

where $\phi \neq 0$ and $\theta \neq 0$.

A) (2p) For the general format ARIMA(p, d, q) replace p, d, q with the integers such that the abbrevation corresponds to the process above.

B) (2p) For what values of ϕ and θ is the process stationary?

C) (2p) For what values of ϕ and θ is the process invertible?

D) (3p) For $\phi = 0.9$ and $\theta = 0.9$, sketch the correlogram of the process, that is, the autocorrelation function and the partial autocorrelation function. No need to derive the ACF or the PACF, just sketch the correlogram. Use as many lags as you find appropriate for the 'pattern' to be obvious.

E) (4p) Write down and solve the characteristic equation for the MA part of the model. For what values of the root is the process invertible?

F) (4p) Now, assume that $\phi = 0$ and $\theta \neq 0$ and derive the infinite autoregressive representation. State any assumption(s) *when* you need them and also state exactly *why* you need them. The end result, that is, the infinite autoregressive repressentation, must be a stationary (causal) representation.

Task 3

(27 points in total)

Consider

$$\phi(B)\,\Phi(B)\,Y_t = \theta(B)\,\Theta(B)\,e_t$$

where $e_t \sim NID(0, \sigma^2)$,

$$\phi(B) = \left(1 - \phi B^1\right),$$
$$\Phi(B) = \left(1 - \Phi B^4\right),$$
$$\theta(B) = \left(1 - \theta B^1\right)$$
$$\Theta(B) = \left(1 - \Theta B^4\right)$$

IMPORTANT: for subtasks A through F (that is A,B,C,D,E,F), assume that $\Phi = \Theta = \phi = 0$, but θ being non-zero.

A) (4p) Derive the expected value of the process. State explicitly any assumption(s) you need, when you need them in order to derive this result. Also, state any assumption(s) needed, if any, for the expected value to exist.

B) (4p) Derive the variance of the process. State explicitly any assumption(s) you need, when you need them in order to derive this result. Also, state any assumption(s) needed, if any, for the variance to exist.

C) (4p) Derive the first autocovariance of the process. State explicitly any assumption(s) you need, if any, when you need them in order to derive this result. Also, state any assumption(s) needed, if any, for the autocovariance to exist.

Let I_t denote the information set of all available information up to and including time t.

D) (3p) Derive the forecast one step out in time by calculating the conditional expected value of the process at time $t+1$ given all the information up to and including time t.

E) (3p) Derive the expected value of the forecast error, (of course, you first need to derive the forecast error).

F) (3p) Derive the forecast error variance.

G) (3p) Now, assume that $\phi = \theta = \Phi = 0$, but $\Theta = 0.9$. Sketch the correlogram, that is, the autocorrelation function and the partial autocorrelation function for the process. No need to derive the ACF or the PACF, just sketch the correlogram. Use as many lags as you find appropriate for the 'pattern' to be obvious.

H) (3p) Now, assume that $\phi = \theta = \Theta = 0$, but $\Phi = 0.9$. Sketch the correlogram, that is, the autocorrelation function and the partial autocorrelation function for the process. No need to derive the ACF or the PACF, just sketch the correlogram. Use as many lags as you find appropriate for the 'pattern' to be obvious.

Task 4

(25 points in total)

Consider the US Real Gross Domestic Product (GDP) (billions of Chained 2009 Dollars), quarterly data (seasonally adjusted) for the period 1947 Q1 to 2016 Q4. For graphs and other Eviews ouput, see Figures below.

A) (6p) Test whether the original data contains a unit root or not. Use the significance level of 5%. Make sure all the steps and calculations you make are easy to follow and understand. Fully document the test procedure as outlined in the test-template.

A high ranking US official, one Mr D. Umb is trying to make sense of this data. He has no idea how to approach it, and thus asks a new found friend, Mr P. Uttin, for help. Mr Uttin says that the the numbers are way to large (and thus cannot be true - most likely the previous secretary of state made them up) and that he should take first difference to make them smaller. See output below for a graph and correlogram of the first difference.

However, a data analyst (not related to Mr Umb and not yet fired) gets hold of this data and runs some analysis of the data and fits an AR(1) of the first difference of GDP.

B) (2p) For the residuals from the AR(1) model: interpret the second (estimated) autocorrelation coefficient.

C) (2p) For the residuals from the AR(1) model: interpret the second (estimated) partial autocorrelation coefficient.

D) (6p) Perform a test to test whether the second, and only the second autocorrelation of the residuals from the AR(1) model is zero. Use the significance level of 5%. Make sure all the steps and calculations you make are easy to follow and understand. Fully document the test procedure as outlined in the test-template.

The data analysts move on and fits an AR(2) to the first difference of the GDP.

E) (6p) Perform a test whether the first three autocorrelations of the residuals from the AR(2) model of the first difference are simultaneously zero. Use the significance level of 5%. Make sure all the steps and calculations you make are easy to follow and understand. Fully document the test procedure as outlined in the test-template.

A rumour emerges in the Grey House that there is someone that knows what he is doing, and our data analysts is being tracked down and fired for being competent. However, before he got fired, the data analyst succeded to run a few more models. Now, an unnamed source in the Grey House leaks these results to the Fence Street Journal and the editor calls you to comment on the results.

F) (3p) Using all the ouput in this task, argue what model that is the 'best'. Use all relevant evaluation criteria to do so. Even if more than one model would be 'ok', pick one and argue for that model. There is no need to do any formal testing in this subtask, but refer to the output when arguing your case.

Epilogue:

In a high level summit Mr Umb and Mr Uttin gets together to discuss Twitter accounts and the state of the world economy. Mr Uttin tells Mr Umb that according his analysis, the US economy is completely flawed and that they would be better off adapting an alternative economic system, that he claims has worked like a charm whereever it has been implemented. Mr Uttin uses the metaphor that if you want to make the case to build a new house, the easiest way to set the stage for that, is to burn the old house to the ground - then you have no other choice than to build a new one. Mr Umb does not really understand this metaphor, but after a few glasses of vodka he promises to do his very best to pave the way for such a transition. How this works out is still to be seen.

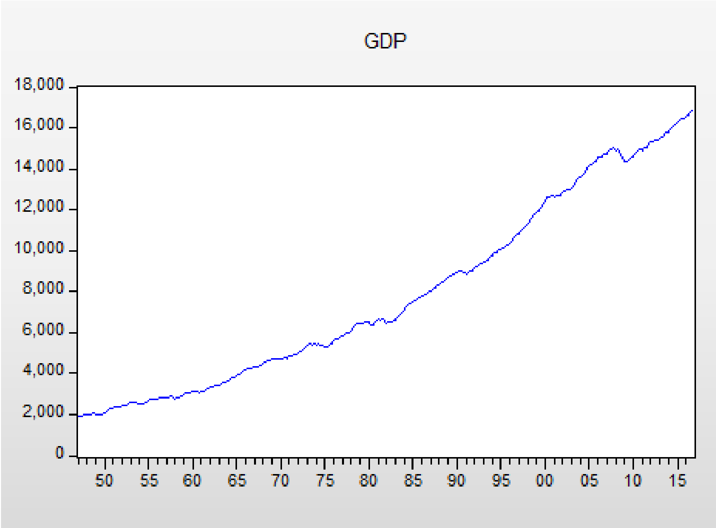
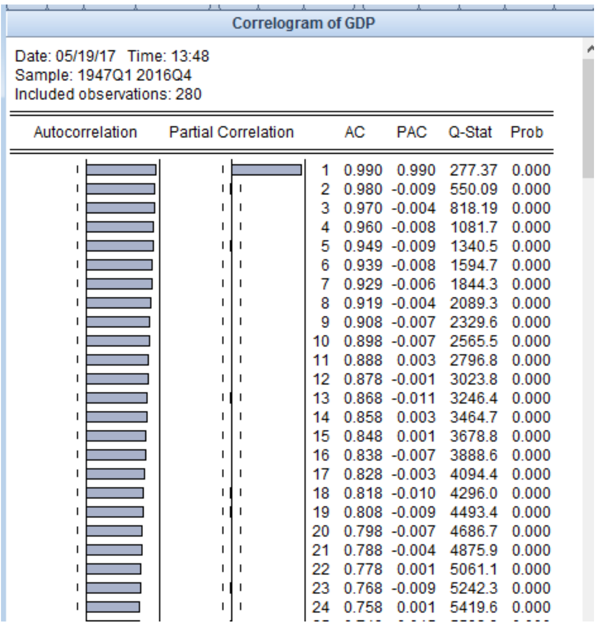


Figure 4.1: Level of US GDP Q1 1947 to Q4 2016.



Correlogram of GDP

Date: 05/19/17 Time: 13:48
Sample: 1947Q1 2016Q4
Included observations: 280

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.990	0.990	277.37	0.000
		2	0.980	-0.009	550.09	0.000
		3	0.970	-0.004	818.19	0.000
		4	0.960	-0.008	1081.7	0.000
		5	0.949	-0.009	1340.5	0.000
		6	0.939	-0.008	1594.7	0.000
		7	0.929	-0.006	1844.3	0.000
		8	0.919	-0.004	2089.3	0.000
		9	0.908	-0.007	2329.6	0.000
		10	0.898	-0.007	2565.5	0.000
		11	0.888	0.003	2796.8	0.000
		12	0.878	-0.001	3023.8	0.000
		13	0.868	-0.011	3246.4	0.000
		14	0.858	0.003	3464.7	0.000
		15	0.848	0.001	3678.8	0.000
		16	0.838	-0.007	3888.6	0.000
		17	0.828	-0.003	4094.4	0.000
		18	0.818	-0.010	4296.0	0.000
		19	0.808	-0.009	4493.4	0.000
		20	0.798	-0.007	4686.7	0.000
		21	0.788	-0.004	4875.9	0.000
		22	0.778	0.001	5061.1	0.000
		23	0.768	-0.009	5242.3	0.000
		24	0.758	0.001	5419.6	0.000

Figure 4.2: Correlogram of US GDP

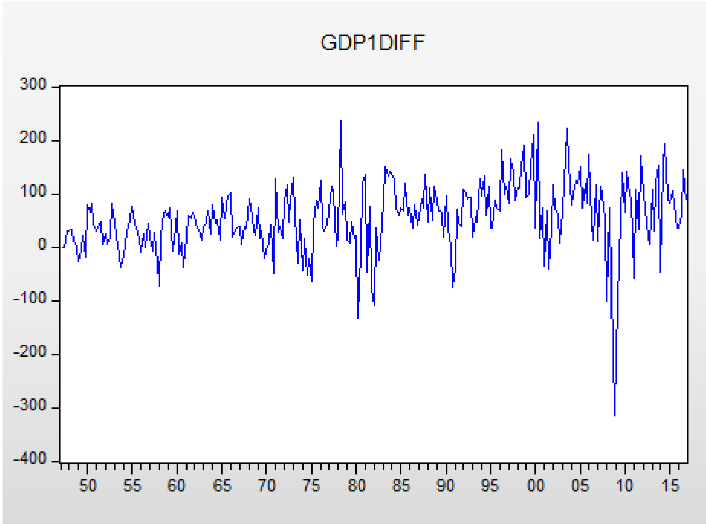
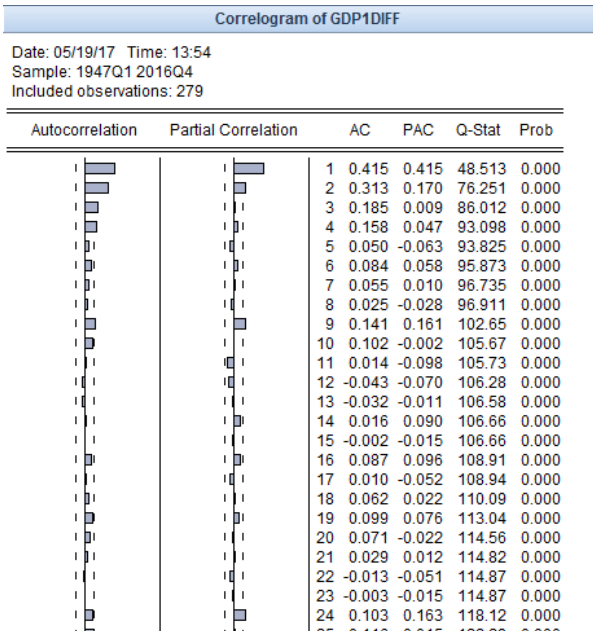


Figure 4.3: First difference of GDP



Correlogram of GDP1DIFF

Date: 05/19/17 Time: 13:54
Sample: 1947Q1 2016Q4
Included observations: 279

	AC	PAC	Q-Stat	Prob
1	0.415	0.415	48.513	0.000
2	0.313	0.170	76.251	0.000
3	0.185	0.009	86.012	0.000
4	0.158	0.047	93.098	0.000
5	0.050	-0.063	93.825	0.000
6	0.084	0.058	95.873	0.000
7	0.055	0.010	96.735	0.000
8	0.025	-0.028	96.911	0.000
9	0.141	0.161	102.65	0.000
10	0.102	-0.002	105.67	0.000
11	0.014	-0.098	105.73	0.000
12	-0.043	-0.070	106.28	0.000
13	-0.032	-0.011	106.58	0.000
14	0.016	0.090	106.66	0.000
15	-0.002	-0.015	106.66	0.000
16	0.087	0.096	108.91	0.000
17	0.010	-0.052	108.94	0.000
18	0.062	0.022	110.09	0.000
19	0.099	0.076	113.04	0.000
20	0.071	-0.022	114.56	0.000
21	0.029	0.012	114.82	0.000
22	-0.013	-0.051	114.87	0.000
23	-0.003	-0.015	114.87	0.000
24	0.103	0.163	118.12	0.000

Figure 4.4: Correlogram of first difference of GDP

Dependent Variable: GDP1DIFF
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 05/19/17 Time: 13:55
Sample: 1947Q2 2016Q4
Included observations: 279
Convergence achieved after 13 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	53.27202	6.196027	8.597771	0.0000
AR(1)	0.414751	0.045333	9.149014	0.0000
SIGMASQ	3477.533	196.3748	17.70866	0.0000

R-squared	0.172639	Mean dependent var	53.32903
Adjusted R-squared	0.166643	S.D. dependent var	64.94829
S.E. of regression	59.29024	Akaike info criterion	11.01414
Sum squared resid	970231.8	Schwarz criterion	11.05318
Log likelihood	-1533.472	Hannan-Quinn criter.	11.02980
F-statistic	28.79530	Durbin-Watson stat	2.139183
Prob(F-statistic)	0.000000		

Inverted AR Roots	.41

Figure 4.5: Estimation output AR(1) fitted to first difference of GDP

Date: 05/21/17 Time: 11:27
Sample: 1947Q1 2016Q4
Included observations: 279
Q-statistic probabilities adjusted for 1 ARMA term

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.071	-0.071	1.4186	
		2	0.144	0.140	7.2767	0.007
		3	0.027	0.047	7.4798	0.024
		4	0.105	0.092	10.617	0.014
		5	-0.051	-0.049	11.347	0.023
		6	0.067	0.033	12.631	0.027
		7	0.023	0.036	12.783	0.047
		8	-0.063	-0.080	13.928	0.052
		9	0.135	0.129	19.254	0.014
		10	0.068	0.094	20.588	0.015
		11	-0.007	-0.028	20.601	0.024
		12	-0.055	-0.080	21.477	0.029
		13	-0.030	-0.081	21.746	0.040
		14	0.041	0.064	22.247	0.052
		15	-0.056	-0.029	23.166	0.058
		16	0.120	0.106	27.471	0.025
		17	-0.061	-0.019	28.578	0.027
		18	0.032	-0.018	28.890	0.036
		19	0.073	0.079	30.519	0.033
		20	0.036	0.004	30.912	0.041
		21	0.014	0.038	30.971	0.056
		22	-0.032	-0.030	31.287	0.069
		23	-0.048	-0.095	31.986	0.078
		24	0.087	0.117	34.305	0.061

Figure 4.6: Correlogram for residuals from AR(1) fitted to first difference of GDP.

Dependent Variable: GDP1DIFF
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 05/19/17 Time: 15:34
Sample: 1947Q2 2016Q4
Included observations: 279
Convergence achieved after 19 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	53.28830	7.424627	7.177236	0.0000
AR(1)	0.343410	0.046134	7.443773	0.0000
AR(2)	0.172922	0.049528	3.491395	0.0006
SIGMASQ	3373.168	187.2456	18.01468	0.0000

R-squared	0.197469	Mean dependent var	53.32903
Adjusted R-squared	0.188714	S.D. dependent var	64.94829
S.E. of regression	58.49985	Akaike info criterion	10.99105
Sum squared resid	941114.0	Schwarz criterion	11.04311
Log likelihood	-1529.252	Hannan-Quinn criter.	11.01194
F-statistic	22.55525	Durbin-Watson stat	2.000103
Prob(F-statistic)	0.000000		

Inverted AR Roots	.62	-.28

Figure 4.7: Estimation output AR(2) fitted to first difference of GDP.

Date: 05/21/17 Time: 11:29
Sample: 1947Q1 2016Q4
Included observations: 279
Q-statistic probabilities adjusted for 2 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.001	-0.001	0.0005	
		2	-0.011	-0.011	0.0376	
		3	-0.001	-0.001	0.0377	0.846
		4	0.056	0.056	0.9239	0.630
		5	-0.067	-0.067	2.2004	0.532
		6	0.044	0.045	2.7498	0.601
		7	-0.001	-0.003	2.7501	0.738
		8	-0.074	-0.077	4.3409	0.631
		9	0.140	0.151	10.063	0.185
		10	0.098	0.087	12.869	0.116
		11	-0.019	-0.013	12.976	0.164
		12	-0.085	-0.077	15.074	0.129
		13	-0.037	-0.066	15.476	0.162
		14	0.028	0.045	15.710	0.205
		15	-0.036	-0.036	16.090	0.244
		16	0.107	0.108	19.479	0.147
		17	-0.060	-0.045	20.557	0.152
		18	0.014	0.002	20.616	0.194
		19	0.085	0.074	22.795	0.156
		20	0.046	0.006	23.423	0.175
		21	0.004	0.053	23.428	0.219
		22	-0.065	-0.055	24.725	0.212
		23	-0.065	-0.083	26.024	0.206
		24	0.095	0.125	28.803	0.151

Figure 4.8: Correlogram for residuals from AR(2) fitted to first difference of GDP.

Dependent Variable: GDP1DIFF
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 05/19/17 Time: 15:25
Sample: 1947Q2 2016Q4
Included observations: 279
Convergence achieved after 15 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	53.26592	7.857911	6.778635	0.0000
AR(1)	0.706264	0.081602	8.655027	0.0000
MA(1)	-0.357212	0.102817	-3.474237	0.0006
SIGMASQ	3379.222	187.6402	18.00905	0.0000

R-squared	0.196028	Mean dependent var	53.32903
Adjusted R-squared	0.187258	S.D. dependent var	64.94829
S.E. of regression	58.55232	Akaike info criterion	10.99283
Sum squared resid	942802.9	Schwarz criterion	11.04489
Log likelihood	-1529.500	Hannan-Quinn criter.	11.01372
F-statistic	22.35063	Durbin-Watson stat	2.016026
Prob(F-statistic)	0.000000		

Inverted AR Roots	.71
Inverted MA Roots	.36

Figure 4.9: Estimation output ARMA(1,1) fitted to first difference of GDP.

Date: 05/21/17 Time: 11:32
Sample: 1947Q1 2016Q4
Included observations: 279
Q-statistic probabilities adjusted for 2 ARMA terms

Lag	AC	PAC	Q-Stat	Prob
1	-0.009	-0.009	0.0247	
2	0.038	0.038	0.4259	
3	-0.036	-0.035	0.7901	0.374
4	0.038	0.036	1.2099	0.546
5	-0.085	-0.082	3.2888	0.349
6	0.028	0.023	3.5133	0.476
7	-0.007	0.001	3.5294	0.619
8	-0.070	-0.080	4.9437	0.551
9	0.138	0.149	10.489	0.163
10	0.088	0.086	12.731	0.121
11	-0.016	-0.028	12.806	0.172
12	-0.082	-0.076	14.778	0.140
13	-0.048	-0.068	15.454	0.163
14	0.026	0.055	15.657	0.207
15	-0.043	-0.037	16.212	0.238
16	0.107	0.100	19.636	0.142
17	-0.059	-0.040	20.666	0.148
18	0.023	-0.006	20.825	0.185
19	0.080	0.081	22.765	0.157
20	0.042	0.003	23.290	0.180
21	-0.001	0.040	23.290	0.225
22	-0.064	-0.049	24.547	0.219
23	-0.065	-0.083	25.840	0.213
24	0.086	0.126	28.135	0.171

Figure 4.10: Correlogram for residuals from ARMA(1,1) fitted to first difference of GDP.

Dependent Variable: GDP1DIFF
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 05/19/17 Time: 15:26
Sample: 1947Q2 2016Q4
Included observations: 279
Convergence achieved after 35 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	53.30438	7.001165	7.613644	0.0000
MA(1)	0.352428	0.047733	7.383262	0.0000
MA(2)	0.266683	0.054158	4.924202	0.0000
MA(3)	0.145849	0.053867	2.707548	0.0072
MA(4)	0.185314	0.050468	3.671880	0.0003
SIGMASQ	3344.270	203.5810	16.42723	0.0000

R-squared	0.204344	Mean dependent var	53.32903
Adjusted R-squared	0.189771	S.D. dependent var	64.94829
S.E. of regression	58.46170	Akaike info criterion	10.99702
Sum squared resid	933051.5	Schwarz criterion	11.07511
Log likelihood	-1528.084	Hannan-Quinn criter.	11.02835
F-statistic	14.02262	Durbin-Watson stat	1.999639
Prob(F-statistic)	0.000000		

Inverted MA Roots	.31+.58i	.31-.58i	-.48-.45i	-.48+.45i

Figure 4.11: Estimation output MA(4) fitted to first difference of GDP.

Date: 05/21/17 Time: 11:35
Sample: 1947Q1 2016Q4
Included observations: 279
Q-statistic probabilities adjusted for 4 ARMA terms

	AC	PAC	Q-Stat	Prob
1	-0.001	-0.001	0.0004	
2	0.011	0.011	0.0347	
3	0.004	0.004	0.0396	
4	-0.008	-0.008	0.0587	
5	0.015	0.015	0.1255	0.723
6	0.061	0.062	1.2085	0.546
7	-0.005	-0.005	1.2148	0.749
8	-0.063	-0.064	2.3476	0.672
9	0.144	0.145	8.3384	0.139
10	0.084	0.089	10.410	0.108
11	0.001	-0.006	10.410	0.167
12	-0.086	-0.098	12.590	0.127
13	-0.045	-0.042	13.195	0.154
14	0.038	0.051	13.632	0.190
15	-0.032	-0.052	13.937	0.237
16	0.114	0.100	17.837	0.121
17	-0.070	-0.050	19.287	0.114
18	0.012	0.013	19.333	0.153
19	0.102	0.085	22.462	0.096
20	0.030	0.010	22.742	0.121
21	0.010	0.030	22.769	0.157
22	-0.061	-0.049	23.911	0.158
23	-0.053	-0.056	24.774	0.168
24	0.092	0.109	27.384	0.125

Figure 4.12: Correlogram for residuals from MA(4) fitted to first difference of GDP.