Written Examination in Time Series Analysis (B3)

Spring 2017

2017-04-28 13.00-17.00

Venue: Magistern

Dag Hammarskjölds väg 31, Kunskapsskolans aula

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Allowed means of assistance:

1. Pen or **pencil** (recommended) and eraser
2. **Calculators**,
   (a) 'programmable' calculator, e.g. calculator with graphing functions is OK.
   (b) Calculators with blue-tooth are not allowed.
   (c) Calculators with access to internet are not allowed.
   (d) Calcuators with which it is possible to send and recieve messages of any kind are not allowed.
3. **Physical (paper) dictionary** (no electronic dictionary allowed).
   (a) Dictionary must contain *no notes* of any kind.
   (b) Each student must have his/her own dictionary. It is not allowed for students to pass a dictionary between them.
4. **Ruler.**
5. Collection of formulae and Statistical Tables named '*Collection of Formulae and Statistical Tables for the B2-Econometrics and B3-Time Series Analysis courses and exams*', that the student brings to the exam location. The collection must contain no notes whatsoever. This implies that you cannot make any notes during the exam. If checked, any notes in the collection will be assumed to have been made beforehand, and thus subject to investigation concerning deceptive conduct in examination (cheating).
6. Please note that a collection of critical values for the Student´s t, Normal, Chi-square and F-distributions is given in the Appendix of the '*Collection of Formulae and Statistical Tables for the B2-Econometrics and B3-Time Series Analysis courses and exams*'.

7. About degrees of freedom in tests: If, by any chance, the degree of freedom number that you need for a critical value is not in the table, say that you need 125, but there is only 120 and 130 in the table, then choose the lower number of degrees of freedom, that is, in this case 120.

8. Also note that the '*Test template*', that should be used when performing tests, is given in the '*Collection of Formulae and Statistical Tables for the B2-Econometrics and B3-Time Series Analysis courses and exams'.*

That is:

1. NO BOOK (except paper-dictionary) is allowed.

2. NO (student-written) notes are allowed.

3. NO other document than the one 'Collection of Formulae and Statistical Tables for Time Series Exam' is allowed.

Instructions: **Please note the following:**

1. Start with reading through the instructions!

2. Make sure you **follow** the instructions!

3. Start with reading through the exam.

4. You may write your solutions in Swedish or English.

5. If you find something unclear or if you suspect a typo/mistake in any of the tasks - please do not hesitate to contact the staff at the exam-location for them to get in touch with the responsible teacher.

6. Total score is **100** points

   (a) If you want the ECTS grades, please indicate that on the cover page!

   (b) For each task the maximum number of points is given within parenthesis, e.g. (16p in total).

   (c) For each subtask the number of points is given within parenthesis, e.g. (2p)

7. All solutions must be on separate sheets. No solutions on the questionnaire! (If so, they will be disregarded.)

8. Make sure your solutions are: easy to read and easy to understand, that is:

(a) For each task that you solve, please start with a new sheet: after Task 1, start with a blank sheet for Task 2, etc.

(b) Write the *task number* at the top of each page, in the

.....**MIDDLE OF THE PAGE!!!**.....

Like:

.....**TASK 1**.....

- if you write it in the upper left corner, the staple will cover it, and there is no for way for the examinator to know if the text of that sheet belongs to the previous sub-task or what it is. The Examinators will not make any 'qualified guesses' of what is being displayed on any given page. It is the responsibility of the student to make sure that every task and sub-task is easily identifiable.

(c) If you continue a sub-task on the next sheet of paper - indicate that at the top of the page - IN THE MIDDLE OF THE PAGE, like, for example:

.....'Task 1B (cont.)'.....

(d) Please separate each subtask A, B etc with a horizontal line across the sheet

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if they are on the same sheet of paper - that way it will be easy for the examinator to actually see where one subtask ends and next begins.

(e) For examinator readability, it is highly recommended that you use a pencil, (and not a pen), which will allow you to erase and rewrite if you make a mistake. Crossed-over text and corrections using 'tipp-ex' will just cause blurriness and confusion to the examinator.

(f) For examinator readability: Write clearly, that is, letters, mathematical/statistical symbols and numbers should be easy recognizable!! Do not underestimate the correlation between readability and points scored, that is, when readability goes to zero, points scored also goes to zero, no matter your intentions or wheather *you* can read it or not.

(g) Also note that everything that you write will be taken at 'face value'. That is, for example, if you write β_1 the examinator will take that as a β_1 even though you may claim that it is given from the context it should be clear that you meant something else, like β_3. Thus, given this example, writing β_1, and that is not correct in that specific formula or statement, this will lead to subtraction of points, even if you will claim that it is just a typo, and that in another task or subtask, it is clear that you understand the issue.

(h) Please put the sheets in **order**, that is first Task 1, and then Task 2 etc...

9. Please keep the questionaire.
10. Do well!

Task 1

(26 points in total)

A) (2p) What is a stochastic process from a theoretical point of view? Explain using words, no formulae needed.

B) (6p) State the conditions for a stochastic process to be *covariance stationary*. For each condition, state that condition using formulae and also explain in words what it means.

To 'apply' the Box-Jenkins methodology, a nessecary condition is that the series in question is (at least) covariance stationary. If a process is *not* stationary, we need to transform it somehow to make it stationary before we can apply the Box-Jenkins methodology.

Consider the process

$$\phi(B) Y_t = \alpha_0 + \alpha_1 t + \theta(B) e_t,$$

where $\phi(B) = (1 - B\phi)$, $\theta(B) = (1 - B\theta)$ and $e_t \sim NID(0, \sigma^2)$

C) (5p) For what values for the parameters ϕ, α_0, α_1 and θ does this process have a stochastic trend, but not a deterministic trend? Given these parameter values, suggest a transformation such that the transformed process is stationary.

D) (5p) For what values for the parameters ϕ, α_0, α_1 and θ does this process have a deterministic trend, but not a stochastic trend? Given these parameter values, suggest a transformation such that the transformed process is stationary.

E) (8p) State the four stages of the Box-Jenkins methodology. For each stage, elaborate on the *purpose* of that specific stage, also give at least *one* example of a tool/method/statistical test that can be used in that specific stage.

Task 2

(22 points in total)

Consider the following process

$$\phi(B) Y_t = \theta(B) e_t$$

where $e_t \sim NID(0, \sigma^2)$.

Let

$$\phi(B) = (1 - B\phi)$$

where $\phi \neq 0$ and

$$\theta(B) = 1.$$

A) (4p) Write down and solve the characteristic equation for the process above.

B) (2p) For what values of the root is the process stationary? What is the corresponding condition for the parameter ϕ?

C) (4p) Derive the infinite MA representation (MA(∞)) of the process *without* using recursive substitution. State the nessecary assumption concerning ϕ (when you need it) and state explicitly *why* you need this assumption.

D) (4p) Assume that the root of the characteristic equation is one. Write down the resulting process. Assume that the process started at Y_1 and that $Y_0 = 0$ and derive the expected value of the process.

E) (4p) For the process from sub-task D, derive the variance of Y_t, (that is, for a general t).

F) (2p) Again, for the process from sub-task D, sketch the ensamble of the process, that is, many many realizations of the process in the same graph.

G) (2p) Sketch the correlogram (SACF and SPACF) of a realization of the process above where you would have 10 000 observations of Y_t. Use the number of lags you find appropriate.

Task 3

(28 points in total)

Consider the following process

$$\phi(B) Y_t = \theta(B) e_t \tag{1}$$

where $e_t \sim NID(0, \sigma^2)$.

Let

$$\phi(B) = (1 - B\phi) \tag{2}$$

where $\phi = 0$ and

$$\theta(B) = \left(B^0\theta_0 - B^1\theta_1 - B^2\theta_2\right). \tag{3}$$

where $\theta_0 = 1$, $\theta_1 \neq 0$ and $\theta_2 \neq 0$.

A) (4p) Derive the expected value of the process. State explicitly any assumption(s) you need, when you need them in order to derive this result. Also, state any assumption(s) needed, if any, for the expected value to exist.

B) (4p) Derive the variance of the process. State explicitly any assumption(s) you need, when you need them in order to derive this result. Also, state any assumption(s) needed, if any, for the variance to exist.

C) (6p) Derive the first two autocovariances of the process. State explicitly any assumption(s) you need, if any, when you need them in order to derive this result. Also, state any assumption(s) needed, if any, for the autocovariances to exist.

D) (2p) Sketch the correlogram of the process, that is the ACF and the PACF. No need to derive the two, just sketch the correlogram for the process.

E) (4p) Let I_t denote the information set of all available information up to and including time t. Calculate the forecast one step out in time by calculating the conditional expected value of the process at time $t+1$ of the process given all the information up to and including time t.

F) (4p) Calculate the expected value of the forecast error, (of course, you first need to derive the forecast error).

G) (4p) Calculate the forecast error variance.

Task 4

(24 points in total)

A researcher is considering the real data set US GDP 1980 - 2009 quartely data. For relevant output for solve these subtasks below, refer to Figures 4.1-4.3.

A) (6p) Test whether the original data contains a unit root or not. Use the significance level of 5%. Make sure all the steps and calculations you make are easy to follow and understand. Fully document the test procedure as outlined in the test-template.

B) (6p) After some inital analysis the researcher finds it approriate to take first difference and first seasonal difference, that is $\nabla_1\nabla_4 Y_t$. The correlogram of the resulting (transformed) process is given in Figure (4.3) .

For the transformed data $\nabla_1\nabla_4 Y_t$ - Perform a test whether the fourth, and only the fourth autocorrelation is significant different from zero. Use the significance level of 5%. Make sure all the steps and calculations you make are easy to follow and understand. Fully document the test procedure as outlined in the test-template.

C) (6p) For the transformed data $\nabla_1\nabla_4 Y_t$ - Perform a test whether the the first three autocorrelations are simultaneously zero. Use the significance level of 5%. Make sure all the steps and calculations you make are easy to follow and understand. Fully document the test procedure as outlined in the test-template.

D) (3p) For the transformed data $\nabla_1\nabla_4 Y_t$−interpret the fourth sample autocorrelation.

E) (3p) For the transformed data $\nabla_1\nabla_4 Y_t$−interpret the eight sample partial autocorrelation.

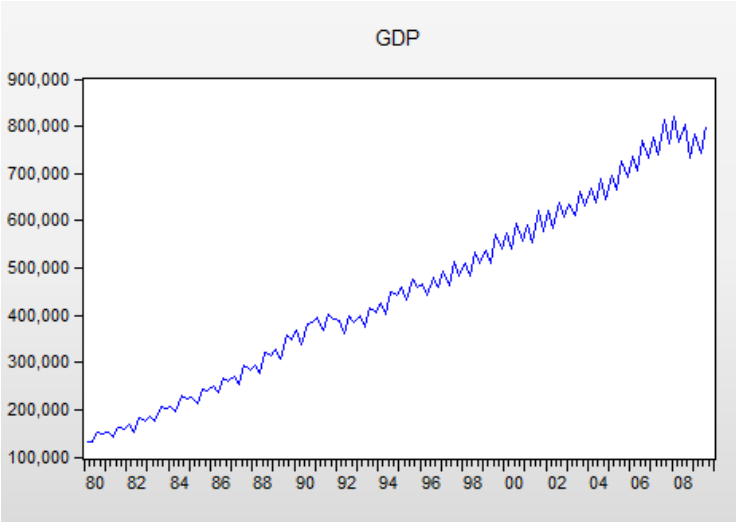


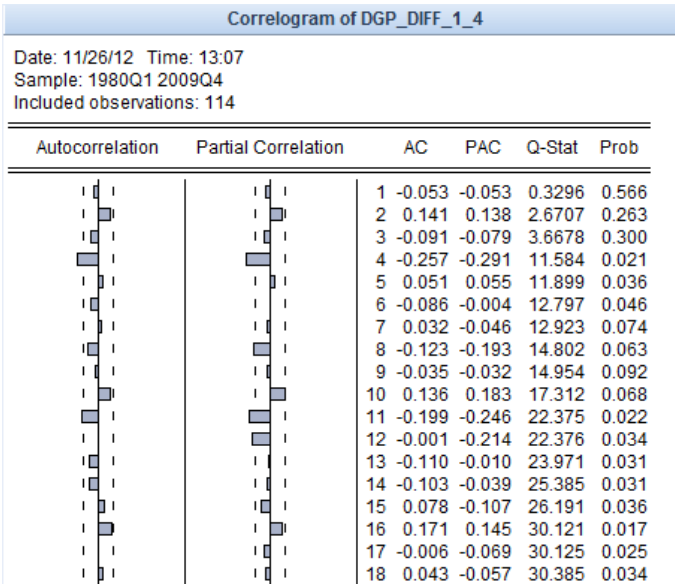
Figure 4.1

Null Hypothesis: GDP has a unit root
Exogenous: Constant
Lag Length: 4 (Automatic - based on SIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.607846	0.8635
Test critical values:	1% level	-3.488585	
	5% level	-2.886959	
	10% level	-2.580402	

*MacKinnon (1996) one-sided p-values.

Figure 4.2



Correlogram of DGP_DIFF_1_4

Date: 11/26/12 Time: 13:07
Sample: 1980Q1 2009Q4
Included observations: 114

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.053	-0.053	0.3296	0.566
		2	0.141	0.138	2.6707	0.263
		3	-0.091	-0.079	3.6678	0.300
		4	-0.257	-0.291	11.584	0.021
		5	0.051	0.055	11.899	0.036
		6	-0.086	-0.004	12.797	0.046
		7	0.032	-0.046	12.923	0.074
		8	-0.123	-0.193	14.802	0.063
		9	-0.035	-0.032	14.954	0.092
		10	0.136	0.183	17.312	0.068
		11	-0.199	-0.246	22.375	0.022
		12	-0.001	-0.214	22.376	0.034
		13	-0.110	-0.010	23.971	0.031
		14	-0.103	-0.039	25.385	0.031
		15	0.078	-0.107	26.191	0.036
		16	0.171	0.145	30.121	0.017
		17	-0.006	-0.069	30.125	0.025
		18	0.043	-0.057	30.385	0.034

Figure 4.3