Uppsala University
Department for Information Science
Statistics

B1

2017-02-17

WRITTEN EXAM FOR THE COURSE, *PROBABILITY THEORY AND STATISTICAL INFERENCE*, B1 (7.5 ECTS)

Writing time: 08^{00}-13^{00}

Permitted aids: Formulas for the course Probability Theory and Statistical Inference
Math-Handout,
Pocket calculator
Dictionary (or word-list)

Notations in the permitted aids are not allowed.

The written examination has 5 problems, for a total of 100 points.

If you desire clarification regarding the test, especially the wording of a problem, then please alert an examination proctor. The examination proctors can contact the responsible instructor.

After turning in your test, you may keep the test-pages with the question-statements.

INSTRUCTIONS:

A. Carefully follow the instructions that are listed on the examination-directions page.

B. State the assumptions that must be made for the method to be applicable.

C. Account for every essential step in your solution. If special concerns are raised in the problem statement, then your solution must carefully address those concerns.

Task 1 (20p)

Five identical bowls are labeled *1*, *2*, *3 4,* and *5*. Bowl ***i*** contains ***i*** white and ***5-i*** black balls, with $i = 1, 2, \cdots, 5$. A bowl is randomly selected and two balls are randomly selected (without replacement) from the contents of the selected bowl.

a) What is the probability that both balls selected are white?
b) Given that both balls selected are white, what is the probability that bowl *3* was selected?

Task 2 (20p)

Given three independent random samples where for sample 1, Y_1 is Bin($n_1 = 4$, $p = 0.2$), for sample 2, Y_2 is Bin($n_2 = 6$, $p = 0.2$) and for sample 3, Y_3 is Bin($n_3 = 10$, $p = 0.2$).

a) What is the probability that Y_1 is larger than one?
b) What is the probability that Y_1, Y_2 and Y_3 all are larger than one?
c) What is the probability that the sum $Z = Y_1 + Y_2 + Y_3$ is larger than three?
d) What is the correlation between Y_1 and Z?

Task 3 (20p)

Given the following function for the random variables Y_1 and Y_2

$$f(y_1, y_2) = \begin{cases} 0.5y_1 + ky_2, & 0 \le y_1 \le 1,\ 0 \le y_2 \le 1 \\ 0 & \text{elsewhere.} \end{cases}$$

a) Determine k so that $f(y_1, y_2)$ becomes a joint density function.

b) Find the marginal density function for Y_1

Task 4 (20)

Let $Y_1, Y_2, \cdots, Y_n$ denote a random sample from the probability density function

$$f(y \mid \theta) = \begin{cases} (\frac{\theta}{2} + 1)y^{\frac{\theta}{2}}, & 0 < y < 1,\ \theta > -2 \\ 0, & \text{elsewhere} \end{cases}$$

a) Find the Maximum likelihood estimator for $\beta = \frac{\theta}{2}$.
b) Find the Maximum likelihood estimator for θ.

Task 5 (20p)

In a survey, a random sample of 1000 individuals was selected. Individuals were asked whether or not they were in favor of a proposal. The number of individuals in favor of the proposal was 460 and the number of individuals not in favor of the proposal was 540. Let p be the proportion in the population that is in favor of the proposal and $1-p$ the proportion not in favor of the proposal.

a) Propose an estimator of the difference between the proportion in favor and the proportion not in favor of the proposal in the population.
b) Calculate a 95% confidence interval for the difference in task a).